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THE RESPONSE OF PRESSURE MEASURING SYSTEMS TO OSCILLATING PRESSURES

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SUMMARY

A method is presented for calculating the response and lag in pressure measuring systems subjected to steady-state sinusoidally varying pressures. The pressure system is assumed to consist of an inlet restriction, tubing length, and connected instrument volume. The material presented is limited by the fact that no theoretical method of predicting the attenuation characteristics of the tubing is given. This limitation is not severe, however, as this characteristic may be experimentally determined for given tube sizes and pressure frequencies.

Experimental data for some sample systems tested are presented and show good agreement with calculated values. The results are presented in such fashion that the qualitative effect of varying the dimensions of system components is apparent. It is therefore possible, once the attenuation characteristics of the tube are determined, to design a system with a required frequency response by a trial-and-error variation of parameters.

INTRODUCTION

Of major interest in many test installations is the response of pressure-distribution systems to rapidly varying pressures, both where these pressures must be accurately measured and where unwanted oscillations must be filtered out or eliminated. When such pressures are measured, the pressure sensing element is normally installed as close to the point of measurement as possible. When this installation is not feasible, connecting tubing must be employed with the consequent possibility of errors caused by resonance or attenuation in the tube. Methods of calculation of the response of pressure systems to small-amplitude steady-state sinusoidal pressures based upon electromechanical analogies have been previously developed (reference 1), but little use has been made of these methods, both because of the large amount of tedious calculation necessary for the solution of even simple pressure systems and because of the lack of information as to whether the equations were valid for pressure oscillations of large amplitude. This paper has been prepared to present a more convenient method of calculating response and lag in pressure systems. The main emphasis has been placed on simplifying the necessary equations to the point where they can be easily applied to practical instrumentation problems. The material consequently is in such form that the qualitative effect of varying the components of a pressure system is easily visualized.

In support of the theoretical methods presented, experimental data have been secured on various pressure systems in the frequency range up to 70 cycles per second. Both the frequency range and pressure systems tested were chosen as being representative of conditions which would be encountered in flight and wind-tunnel installations.

SYMBOLS

a	velocity of propagation, feet per second
A	attenuation factor, seconds$^{1/2}$ per foot
C	capacity, farads
E	voltage, volts
f	frequency, cycles per second
f_o	tube resonant frequency, cycles per second
I	current, amperes
k	ratio of specific heat, c_p/c_v
l	tube length, feet
L	inductance, henries
P_{av}	mean pressure in tube, pounds per square foot
ΔP	pressure difference, pounds per square foot
r	radius, feet
R	resistance, ohms
V	volume, cubic feet
Y	shunt admittance of tube, foot^5 per pound-second
Z	series impedance, pound-seconds per foot^5
Z_o	characteristic impedance of tube, pound-seconds per foot^5

α attenuation constant, 1/feet

β propagation constant, radians per foot

λ wave length, feet

ρ_{av} mean density in tube, slugs per cubic foot

μ coefficient of viscosity, pounds per foot-second

ω angular frequency, radians per second

Subscripts:

d quantity existing at inlet restriction or restriction parameter

r quantity existing at instrument or instrument parameter

S quantity existing at pressure-system inlet

THEORY

General Theory

The measurement of rapidly varying pressures requires in most cases that a pressure instrument be connected to the measuring point through a finite length of connecting tube. The tube opening may be restricted by a connector of smaller opening, either because of aerodynamic considerations or because the response of the measuring system to the oscillating pressures must be adjusted. In most cases exposing the pressure-measuring diaphragm to a reference pressure is necessary. This procedure requires that the diaphragm be installed so that it is exposed to a reference pressure volume which may be connected by means of tubing to a reference pressure source. The reference volume and connecting tubing is hereinafter referred to as the reference pressure system. For the purpose of the following analysis, it will be considered that: (1) the response of the instrument may be separately evaluated or is constant throughout the frequency range, and that (2) deflections of the sensitive element are sufficiently small so that negligible changes in internal volume occur and no energy is transferred to the reference pressure systems.

The air column in a tube has mass inertia, elasticity, and can dissipate energy with its motion; consequently, as is generally known, wave motion can be propagated along its length. The equations governing this motion have been previously derived for small-amplitude pressure variations (references 1 and 2) and are exactly similar to the equations which govern the propagation of electrical waves on transmission lines. As these equations already have been developed, it is relatively easy to describe the behavior of the pressure system in terms of the analogous electrical system by use of the usual electrical notation (reference 3). The electrical terms and the equivalent acoustical terms used herein are shown in the following table:

Electrical			Equivalent acoustical		
Term	Unit	Symbol	Term	Unit	Symbol
inductance	henries	L	inertance	pound-seconds^2 per foot^5	L
capacity	farads	C	volumetric capacity	foot^5 per pound	C
resistance	ohms	R	flow resistance	pound-seconds per foot^5	R
current	amperes	I	volume flow	feet^3 per second	Q
voltage	volts	E	pressure	pounds per foot^2	P

In acoustical terms (reference 4), the inductance per unit length of line $L = \frac{\rho_{av}}{\pi r^2}$, the capacitance per unit length $C = \frac{\pi r^2}{kP_{av}}$, and the resistance per unit length $R = \frac{\Delta P}{Q}$ (the latter being dependent on the type and amplitude of flow).

The behavior of the system can then be defined by the general equations for a transmission line

$$E_S = E_r \cosh \sqrt{ZY}\, l + I_r Z_o \sinh \sqrt{ZY}\, l \tag{1}$$

$$I_S = I_r \cosh \sqrt{ZY}\, l + \frac{E_r}{Z_o} \sinh \sqrt{ZY}\, l \tag{2}$$

where

$$Z = R + j\omega L$$

$$Y = j\omega C$$

The quantity $\sqrt{ZY}$ is a complex number and may therefore be written as

$$\sqrt{ZY} = \alpha + j\beta \tag{3}$$

where α is an attenuation constant determined by the decrement in pressure amplitude per length of tube and β is a propagation constant or phase-angle change per unit length of tube as defined by the following equation:

$$\beta = \frac{2\pi f}{\text{Propagation velocity}}$$

The quantity Z_o is designated the characteristic impedance of the tube

$$Z_o = \sqrt{\frac{Z}{Y}} = \frac{\sqrt{ZY}}{Y} = \frac{\alpha + j\beta}{Y} \tag{4}$$

Equation (1) may be rewritten to give

$$\frac{E_S}{E_r} = \cosh \sqrt{ZY}\ l + \frac{Z_o}{Z_r} \sinh \sqrt{ZY}\ l \tag{5}$$

Substituting equation (3) into equation (5) and simplifying by trigonometric substitutions gives

$$\frac{E_S}{E_r} = \sqrt{\sinh^2\alpha l + \cos^2\beta l}\ \underline{/\tan^{-1}\left(\tan \beta l \tanh \alpha l\right)}$$

$$+ \frac{Z_o}{Z_r}\sqrt{\sinh^2\alpha l + \sin^2\beta l}\ \underline{/\tan^{-1}\left(\frac{\tan \beta l}{\tanh \alpha l}\right)} \tag{6}$$

Equation (6) defines the ratio of the voltage or pressure amplitude at the opén end of the tube to the amplitude existing at the pressure capsule. The reciprocal of this ratio is called herein the response of the system. The right-hand terms in equation (6) are given in polar coordinates and must be added vectorially at the indicated angles.

Simplified Theory

Characteristics of system having negligible instrument volume.- If the pressure capsule is sufficiently small, negligible air flow occurs at the instrument end of the tube, Z_r approaches infinity, and equation (1) reduces to

$$\frac{P_S}{P_r} = \frac{E_S}{E_r} = \cosh \sqrt{ZY}\ l \tag{7}$$

$$\frac{P_S}{P_r} = \left(\sinh^2\alpha l + \cos^2\beta l\right)^{1/2} \underline{/\tan^{-1}(\tan \beta l \tanh \alpha l)} \tag{8}$$

Further, if the tube is of sufficiently large diameter, negligible attenuation of the pressure wave occurs in the tube, α approaches zero, and equation 8 simplifies to

$$\frac{P_S}{P_r} = \cos \beta l \tag{9}$$

At resonance frequencies, equation (9) becomes zero and the response $\frac{P_r}{P_S}$ becomes infinite. The following simple relationship can then be derived for the determination of the resonant frequency:

$$\cos \beta l = 0 \tag{10}$$

$$\beta l = \frac{2\pi f_o l}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots \tag{11}$$

where

$$f_o = \frac{a}{4l}, \frac{3a}{4l}, \frac{5a}{4l} \ldots \tag{12}$$

where f_o is the resonant frequency of the tube with no attached volume. As the wave length of a pressure wave is given by the relation $f\lambda = a$, equation (12) indicates that, at resonance frequencies, the tube length is an odd multiple of 1/4 wave length.

Figure 1 is a plot of response and phase angles based on equations (8) and (9) for simple systems having zero or finite attenuation.

Characteristics of system having an inlet restriction.- In a similar system having negligible instrument volume so that I_r approaches zero, the effect of adding a constriction at the tube inlet may be evaluated as indicated in the following discussion.

For restrictions which are short in length compared to 1 wave length, the flow impedance consists of a resistance caused by viscous pressure losses and an inertance caused by the mass of air in the restriction. As derived in reference 5,

$$Z_d = \frac{l_d}{\pi r_d^2}\left(\frac{8\mu}{r_d^2} + \frac{4}{3}j\omega\rho_{av}\right) \tag{13}$$

This impedance causes a pressure loss,

$$E_S - E_S' = I_S Z_d \tag{14}$$

where E_S is the applied pressure and E_S' is the pressure applied to the tube past the restriction.

From equations (1) and (2), when I_r equals zero

$$E_S' = E_r \cosh \sqrt{ZY}\, l \tag{15}$$

$$I_S = \frac{E_r}{Z_o} \sinh \sqrt{ZY}\, l \tag{16}$$

By the substitution of values from equations (15) and (16) into equation (14),

$$\frac{E_S - E_S'}{E_S'} = \frac{Z_d}{Z_o} \tanh \sqrt{ZY}\, l \tag{17}$$

$$\frac{E_S}{E_S'} = 1 + \frac{Z_d}{Z_o} \tanh(\alpha l + j\beta l) \tag{18}$$

Equation (18) was derived to secure the ratio of the pressure applied to the pressure existing in the tube past the restriction. The magnitude of the over-all response of the tube and restriction can now be secured by multiplying the effectiveness of the restriction, as given by equation (18), by the relation for the tube without the restriction, as given by equation (8).

$$\frac{P_S}{P_r} = \frac{E_S}{E_r} = \frac{E_S'}{E_r}\frac{E_S}{E_S'}$$

$$= \left[1 + \frac{Z_d}{Z_o} \tanh(\alpha l + j\beta l)\right] \left(\sinh^2\alpha l + \cos^2\beta l\right)^{1/2} \underline{/\tan^{-1}(\tan \beta l \tanh \alpha l)} \tag{19}$$

The effectiveness of the restriction can be shown to vary with the applied frequency and the tube characteristics. In order to visualize the effect of the constriction on the response of the system, equation (18) may be rewritten by trigonometric substitution

$$\frac{E_S}{E_S'} = 1 + \frac{Z_d}{Z_o}\left(\frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l}\right) \tag{20}$$

At antiresonance frequencies ($\beta l = 0, \pi, n\pi$), $\tan \beta l = 0$ and equation (20) reduces to

$$\frac{E_S}{E_S'} = 1 + \frac{Z_d}{Z_o} \tanh \alpha l \tag{21}$$

At resonance frequencies $\left(\beta l = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{n\pi}{2}\right)$, $\tan \beta l = \infty$ and equation (20) reduces to

$$\frac{E_S}{E_S'} = 1 + \frac{Z_d}{Z_o} \frac{1}{\tanh \alpha l} \tag{22}$$

For the case of a resonant tube $\tanh \alpha l \ll 1$ and therefore it can be seen from equations (21) and (22) that the restriction is extremely effective in reducing large amplitude resonances but has only a negligible effect at antiresonance frequencies.

In the case of a large-diameter tube wherein the attenuation is negligible, α approaches zero and equation (19) reduces to

$$\frac{P_S}{P_r} = \frac{E_S}{E_r} = \left(1 + \frac{Z_d}{Z_o} \tanh j\beta l\right)\cos \beta l \tag{23}$$

$$\frac{P_S}{P_r} = \cos \beta l + j\frac{Z_d}{Z_o} \sin \beta l \tag{24}$$

The impedance of a small-diameter constriction is almost a pure resistance since the viscous forces which cause pressure losses are much larger than the inertia forces caused by the mass of air in the constriction. The ratio of the impedances Z_d/Z_o therefore closely approaches a real number and equation (24) can be rewritten in polar coordinates to give

$$\frac{P_S}{P_r} = \left[\cos^2\beta l + \left(\frac{Z_d}{Z_o} \sin \beta l\right)^2\right]^{1/2} \underline{/ \tan^{-1}\left(\frac{Z_d}{Z_o} \tan \beta l\right)} \tag{25}$$

When $\dfrac{Z_d}{Z_o} = 1$,

$$\frac{P_S}{P_r} = 1 \underline{/ \beta l} \tag{26}$$

A plot of equation (25) is given in figure 2 for two assumed values of the ratio Z_d/Z_o. As noted previously, the restriction at the tube inlet is extremely effective at resonance frequencies but has no effect upon the response at antiresonance frequencies. The principal difference between an adjustment to the response of a system by means of small-diameter tubing or an inlet restriction may be clearly seen from a comparison of figures 1 and 2. Although it is possible, by use of a large-diameter tube and a suitable inlet restriction, to secure unit response over a large frequency range, the use of a small-diameter tube inherently causes decreased response at higher frequencies.

Characteristics of pressure systems having instrument volumes.- Pressure-measuring instruments which are designed to have uniform response over a wide range of frequencies are necessarily designed with high-frequency, low-deflection-type diaphragms. The instrument impedance in such case is a function only of its volumetric capacity and can be written

$$Z_r = \frac{1}{j\omega C_r} = \frac{kP_{av}}{j\omega V_r} \tag{27}$$

The characteristic impedance of the tube as given by equation (4) is

$$Z_o = \frac{\alpha + j\beta}{Y} = \frac{\left(\alpha l + j\beta l\right)kP_{av}}{j\omega\pi r^2 l} \tag{28}$$

The ratio of these impedances, which appears in equation (6), is

$$\frac{Z_o}{Z_r} = \frac{V_r}{\pi r^2 l}\left[(\alpha l)^2 + (\beta l)^2\right]^{1/2} \underline{/\ \tan^{-1}\frac{\beta}{\alpha}} \tag{29}$$

For a given tube diameter and length the value of equation (29) depends directly on the ratio of the volume of the instrument to the total volume of the tube. Equation (6) may now be altered to include only values of real quantities and phase angles,

$$\frac{P_S}{P_r} = \left(\sinh^2\alpha l + \cos^2\beta l\right)^{1/2} \underline{/\ \tan^{-1}(\tan\beta l\ \tanh\alpha l)} + \frac{V_r}{\pi r^2 l}\left[(\alpha l)^2\right.$$

$$\left. + (\beta l)^2\right]^{1/2}\left(\sinh^2\alpha l + \sin^2\beta l\right)^{1/2} \underline{/\ \underline{/\tan^{-1}\left(\frac{\tan\beta l}{\tanh\alpha l}\right)} + \underline{/\tan^{-1}\left(\frac{\beta}{\alpha}\right)}} \tag{30}$$

In the case of a large-diameter tube in which α approaches zero, equation (30) reduces to

$$\frac{P_S}{P_r} = \cos\beta l - \frac{V_r}{\pi r^2 l}\beta l \sin\beta l \tag{31}$$

A plot of equation (31) is given in figure 3 for two values of the ratio $\frac{V_r}{\pi r^2 l}$. The plot indicates that the resonant frequency of a pressure system decreases with an increase in instrument volume. It also shows that instruments having volumes of the same order of magnitude as the total tube volume cause a significant decrease in response at higher frequencies.

At the resonant frequency of the tube with attached instrument volume, equation (31) may be set equal to zero so that

$$\cos\beta l = \frac{V_r}{\pi r^2 l}\beta l \sin\beta l \tag{32}$$

Then,

$$\frac{V_r}{\pi r^2 l} = \frac{\cot \beta l}{\beta l} \tag{33}$$

If values from equations (11) and (12) are substituted in equation (33),

$$\frac{V_r}{\pi r^2 l} = \frac{\cot\left(\frac{\pi}{2} \frac{f_{resonance}}{f_o}\right)}{\frac{\pi}{2} \frac{f_{resonance}}{f_o}} \tag{34}$$

Equation (34) offers a simple method for estimating the lowest resonance frequency of a tube and volume system. This equation is plotted in figure 4 so that it is possible, if only the physical dimensions of a pressure system with negligible tube attenuation are known, to use this chart to determine the resonant frequency of the system.

Effect of appreciable instrument deflections and reference pressure systems on the response of pressure systems.- The analysis of the response of a pressure system when the pressure diaphragm is sufficiently deflected so that it can transmit energy into a reference pressure system is considered beyond the scope of this work. Although the effect on the response is small in most cases, experimental evidence of the character of these effects is shown herein.

LIMITATIONS OF THEORY

Numerical solutions of the equations presented herein can be secured if the parameters β and α are known. The value of β can be calculated from the velocities of propagation plotted in figure 5. This figure is based on the Rayleigh formula for propagation in tubes (reference 2) and, for ease of computation, upon a velocity of propagation of 1000 feet per second in free air. Values of α have been calculated by various investigators for sound pressure amplitudes; however, it is difficult to predict its value for large pressure amplitudes since steady-state laminar flow does not exist in the tube. Reference 6 presents a semiempirical equation which indicates that the attenuation constant α varies with the following factors:

(a) Directly as the square root of the applied frequency

(b) Inversely as the square root of the mean density of the fluid

(c) Inversely with the tube diameter

(d) Directly as the square root of the "effective viscosity." The "effective viscosity" is shown to depend upon the Reynolds number of the flow in the tube, which in turn is directly dependent upon pressure amplitude and frequency.

The effects of factors (a) and (c) on the values of α have been checked by the experimental data presented; however, lack of suitable equipment has made it impossible to generate large-amplitude pressures at various mean densities to substantiate factors (b) and (d).

The lack of any method for calculating the attenuation constant directly limits the general application of the preceding equations. An experimental determination of α is possible, however, by making measurements on a simple system (long tube with no restriction and negligible instrument volume) and then applying the experimentally determined value to the calculation of more complicated systems. For the range of pressure amplitudes and frequencies covered in this investigation, values of α have been determined experimentally and the results are given in the section entitled "Experimental Investigation."

It is important to note that in many practical applications wherein the primary consideration is an estimate of resonance and antiresonance frequencies, sufficient accuracy can be secured by assuming that the attenuation constant α is negligible. In such cases, the equations presented for tube systems having zero attenuation can be applied with consequent reduction of computation time.

EXPERIMENTAL INVESTIGATION

Apparatus and Tests

A schema of the test setup is given in figure 6. The pressure source used in these tests consisted of a piston, driven by a variable-speed electric motor, in a cylinder surrounded by a clearance volume. Adjustment of pressure amplitude was achieved by varying the clearance volume or altering the stroke of the crank and connecting-rod mechanism driving the piston. Directly connected to the cylinder was a standard NACA mechanical-optical pressure recorder, which was used as a pressure standard. This instrument consists of a corrugated diaphragm assembly having an internal volume of 0.2 cubic inch surrounded by a reference volume of approximately 1.2 cubic inches. Deflections of the diaphragm are converted by means of a bell-crank tilting-mirror linkage into deflections of a record line on a photographic film. The natural frequency of this instrument was sufficiently high to require no corrections for its response. Another connection from the pressure generator led to the pressure system under test. The pressure systems tested consisted of various lengths of neoprene pressure tubing varying in diameter from $\frac{1}{8}$-inch to $\frac{3}{16}$-inch inside diameter with connected restrictions and volumes.— The pressure generator supplied oscillating pressure amplitudes up to $\pm$ 10 inches of water at frequencies ranging from 0 to 70 cycles per second. Records of the generated pressure as determined by the reference-pressure cell, pressures existing in the test instrument, and $\frac{1}{10}$-second timing marks were all recorded on the same film. The tests were made in the Flight Instrument Development Section of the Langley Instrument Research Division.

Results of Amplitude Response Tests

Simple tube system with negligible instrument volume.- Figures 7 and 8 summarize the results of tests made with $\frac{1}{8}$-inch and $\frac{3}{16}$-inch-inside-diameter tubes with applied pressure amplitudes of $\pm$ 10 inches of water. The length of tube, given in wave lengths, is calculated from the velocity of propagation as given in figure 6 and the relation a/f equals wave length. In figure 7 the response of systems using $\frac{1}{8}$-inch-inside-diameter tubing is seen to be such that large attenuation of pressure amplitude occurs in the main portion of the frequency range up to 70 cycles per second. Figure 8 indicates that the attenuation in $\frac{3}{16}$-inch-inside-diameter tubes is small enough so that, with suitable damping of the resonance peaks, the response through a large frequency range can be made to approximate unity. Tests made on other tube lengths not shown in these figures fair in well with the plotted curves.

Based upon figures 7 and 8 and equation (8), the attenuation constant α was determined for both tube diameters. The attenuation constant was found to vary with the square root of the applied frequency.

Values of the attenuation factor A are plotted in figure 9. The value of A as calculated from equation (8) is apparently not constant for the shorter tube lengths; however, this effect is actually caused by the finite volume of the pressure capsule. The values of A asymptotically approach their true value for the longer tubes since the attenuation in the tube becomes the determining factor in the over-all response. The values of α thus determined are as follows:

For $\frac{1}{8}$-inch-inside-diameter tubes,

$$\alpha = 0.014\sqrt{f}$$

For $\frac{3}{16}$-inch-inside-diameter tubes,

$$\alpha = 0.0065\sqrt{f}$$

The curves of figures 10 and 11 have been calculated on the basis of the attenuation factor for the $\frac{3}{16}$-inch-inside-diameter tube. The variation between the calculated response curves for zero instrument volume and for 0.21 cubic inch instrument volume shows the effect of the volume of the instrument used (fig. 10). The experimental points for the 10-foot tube with an instrument volume of 0.21 cubic inch approximate a theoretical curve for a 10-foot tube with no volume attached which has an attenuation constant equal to 0.01. The comparison shown in figure 10 between these experimental points and the theoretical curve for zero volume indicates that except at resonance frequencies a relatively large variation in the attenuation factor causes only minor changes in the general characteristics of the response curve.

Tube with inlet restriction.- Figure 11 illustrates the correlation between the calculated response curves and the experimental data for a 10-foot tube with and without an inlet restriction subjected to pressure amplitudes of $\pm$ 10 inches of water. The damping restriction, as previously indicated, is placed at the open tube end. The main effect of the damper at resonance frequencies and the almost negligible effect at antiresonance frequencies should be noted on these curves. Inasmuch as the experimental data for a $\frac{1}{16}$-inch-inside-diameter, $\frac{1}{4}$-inch-long connector seem to correspond more exactly to the calculated values for a connector of twice this length, the losses in this connector can be assumed to be larger than those predicted by equation (13). These added losses are attributed to the inlet and exit losses of the connector and to the fact that steady laminar flow does not exist in the connector.

Tube with appreciable instrument volume.- Figure 10 also indicates the response to sinusoidal pressure amplitudes of $\pm$ 10 inches of water of a 10-foot length of $\frac{3}{16}$-inch-inside-diameter tubing with volumes of 3.05 cubic inches and 6.1 cubic inches added adjacent to the recording instrument. The correlation between calculated and experimental curves indicates that although a high percentage accuracy has not been achieved, good agreement exists insofar as response-curve shape and attenuation characteristics are concerned. The effect of increasing the recording instrument volume is seen to be a lowering of the resonant frequency of the system and a decrease in amplitude of the recorded pressures throughout most of the frequency range.

Effect of some reference pressure systems on the response characteristics of a pressure system.- When pressure recorders are connected to both a pressure measuring system and a reference pressure system, appreciable interaction and energy transfer may occur, which can alter significantly the response of the entire system. The calculation of the response of such systems is considered inadvisable since it is necessarily tedious and the accuracy is questionable. Figure 12 is included as representative of the interactions which occurred with the capsule employed in these tests in a test setup designed so that the interaction was very pronounced. The variations in the response curves are typical of coupled systems which may be encountered in practice. The equivalent pressure and electrical systems are schematically shown in the same figure. It should be noted that these effects may be decreased or eliminated by enlarging the reference volume surrounding the measuring element so that its equivalent electrical capacity becomes extremely large and approaches a short circuit.

Results of Phase-Shift Determination

Figure 13 illustrates the correlation between calculated and experimental lag curves for 10 feet of $\frac{3}{16}$-inch-inside-diameter tubing with various added volumes. The response curves of figure 10 show that the following general characteristics are common to the lag curves of figure 13:

(1) The phase angle shifts relatively slowly until a resonance frequency is reached, at which time the phase changes rapidly through 90°.

(2) The lag remains almost constant at approximately 180° from frequencies above resonance through the first antiresonance frequency and then increases to larger values.

(3) The rate of change of lag angle with increasing frequency becomes more and more linear as the magnitude of the amplitude response at resonance becomes smaller and smaller.

Sample Calculation

Equation (20) has been presented in such form that the response and lag may be arithmetically calculated. Table 1 indicates the calculations necessary for the determination of the response of 10 feet of $\frac{3}{16}$-inch-inside-diameter tubing with an added volume of 0.61 cubic inch. All computations are arithmetic except that for column 20, which may be done graphically with little labor. The determination of the response for various other added volumes can be easily made by recalculating columns 6, 19, and 20 only.

CONCLUSIONS

A method has been developed for estimating the dynamic response of pressure systems subjected to steady-state oscillating pressures which can be applied to the design of these systems either to secure good response over a desired frequency range or to eliminate unwanted resonances when only a mean pressure level is desired. Although no method of predicting the attenuation constant of various tubing under all pressure conditions has been presented, this attenuation constant may be determined experimentally in a simple tube system and used for the design of other more complicated systems.

Even in long tubes of small diameter (that is, 20 ft of $\frac{3}{16}$-in.-I.D. tubes), resonances can occur which cannot be ignored in the interpretation of recorded data. The resonance frequency range for tubes of approximately this length is the frequency range in which airplane buffeting may occur and airplanes passing through contiguous atmospheric gusts may also be subjected to pressure cycles in this range. The direct interpretation of such recorded data without reference to the effect of the recording system will lead to erroneous results.

It can be concluded from the material presented that for accurate dynamic-pressure measurements the first resonant frequency of the pressure-measurement system should be kept well above the highest pressure frequency to be measured. This result can usually be accomplished only by installing the pressure sensing element as close to the point of measurement as possible. When such installation is not feasible, the principles presented in this paper should be applied to the design of an appropriate pressure system. The errors inherent in such a method should be mitigated whenever possible by a direct calibration under conditions of use.

Langley Aeronautical Laboratory
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REFERENCES

1. Mason, Warren P.: Electromechanical Transducers and Wave Filters. D. Van Nostrand Co., Inc., 1942, pp. 106-118.

2. Rayleigh, (Lord): The Theory of Sound. Second ed., vol. II, Macmillan & Co., Ltd. (London), 1896. (Reprinted 1929.)

3. Everitt, William Littell: Communication Engineering. McGraw-Hill Book Co., Inc., 1937, pp. 94-177.

4. Olson, Harry F.: Dynamical Analogies. D. Van Nostrand Co., Inc., 1943.

5. Crandall, Irving B.: Theory of Vibrating Systems and Sound. D. Van Nostrand Co., Inc., 1926, appendix A.

6. Binder, R. C.: The Damping of Large Amplitude Vibrations of Fluid in a Pipe. Jour. Acous. Soc. Am., vol. 15, no. 1, July 1943, pp. 41-43.

TABLE I

RESPONSE OF 10 FEET OF $\frac{3}{16}$- INCH-INSIDE-DIAMETER TUBING WITH 0.61 CUBIC INCH ADDED VOLUME

$$\left[\frac{P_s}{P_r} = \left[\sinh^2 \alpha l + \cos^2 \beta l\right]^{1/2} \underline{/\tan^{-1}(\tan \beta l + \tanh \alpha l)} + \frac{V_r}{\pi r^2}\left[\alpha^2 + \beta^2\right]^{1/2}\left[\sinh^2 \alpha l + \sin^2 \beta l\right]^{1/2} \underline{\Big/\left(\tan^{-1}\frac{\beta}{\alpha} + \tan^{-1}\frac{\tan \beta l}{\tanh \alpha l}\right)}\right]$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	
f	$\alpha = 0.0065\sqrt{f}$	$\frac{2\pi f}{a}$	$((2)^2+(3)^2)^{1/2}$	$\mathrm{Tan}^{-1}\frac{(3)}{(2)}$	$\frac{V_r}{\pi r^2}\times(4)$	Sinh (2) l	Cosh (2) l	Tanh (2) l	Cos (3) l	Tan (3) l	$((7)^2+(10)^2)^{1/2}$	(9) × (11)	Tan^{-1} (13)	$((8)^2-(10)^2)^{1/2}$	$\frac{(11)}{(9)}$	Tan^{-1} (16)	$\angle(17)+(5)$	(6) × (15)	$(12)\angle(14) + (19)\angle(18)$	$\frac{1}{(20)} = \frac{P_r}{P_s}$	a, fig. 2
5	0.0145	0.045	0.047	72.	0.086	0.145	1.01	0.144	0.90	0.50	0.91	0.07	4.1	0.46	3.47	74.	146	0.04	0.89	1.13	700
10	.0205	.083	.086	76.	.157	.206	1.02	.202	.674	1.1	.70	.22	12.4	.77	5.48	80.	156	.12	.62	1.61	760
15	.0252	.118	.120	78.	.220	.254	1.03	.250	.381	2.4	.46	.60	31.0	.97	9.60	84.	162	.21	.36	2.77	800
20	.0291	.151	.153	79.	.282	.295	1.04	.283	.061	16.4	.30	4.64	77.8	1.04	58.0	89.	168	.29	.42	2.40	830
30	.0356	.219	.220	81.	.403	.361	1.06	.342	-.581	-1.4	.69	-.48	154	.88	-4.1	104.	184	.36	1.10	.91	860
40	.0411	.286	.288	82.	.529	.422	1.09	.389	-.961	-.30	1.05	-.12	173	.51	-.77	142.	224	.27	1.23	.81	880
50	.0460	.353	.357	82.	.651	.476	1.11	.430	-.925	.40	1.05	.17	190	.62	.93	223.	305	.41	.95	1.05	890
55	.0482	.384	.387	83.	.708	.500	1.12	.450	-.766	.80	.91	.36	200	.82	1.77	240.	323	.58	.78	1.29	900
60	.0504	.419	.422	83.	.772	.525	1.13	.465	-.498	1.7	.72	.79	218	1.02	3.66	255.	338	.79	.77	1.31	900

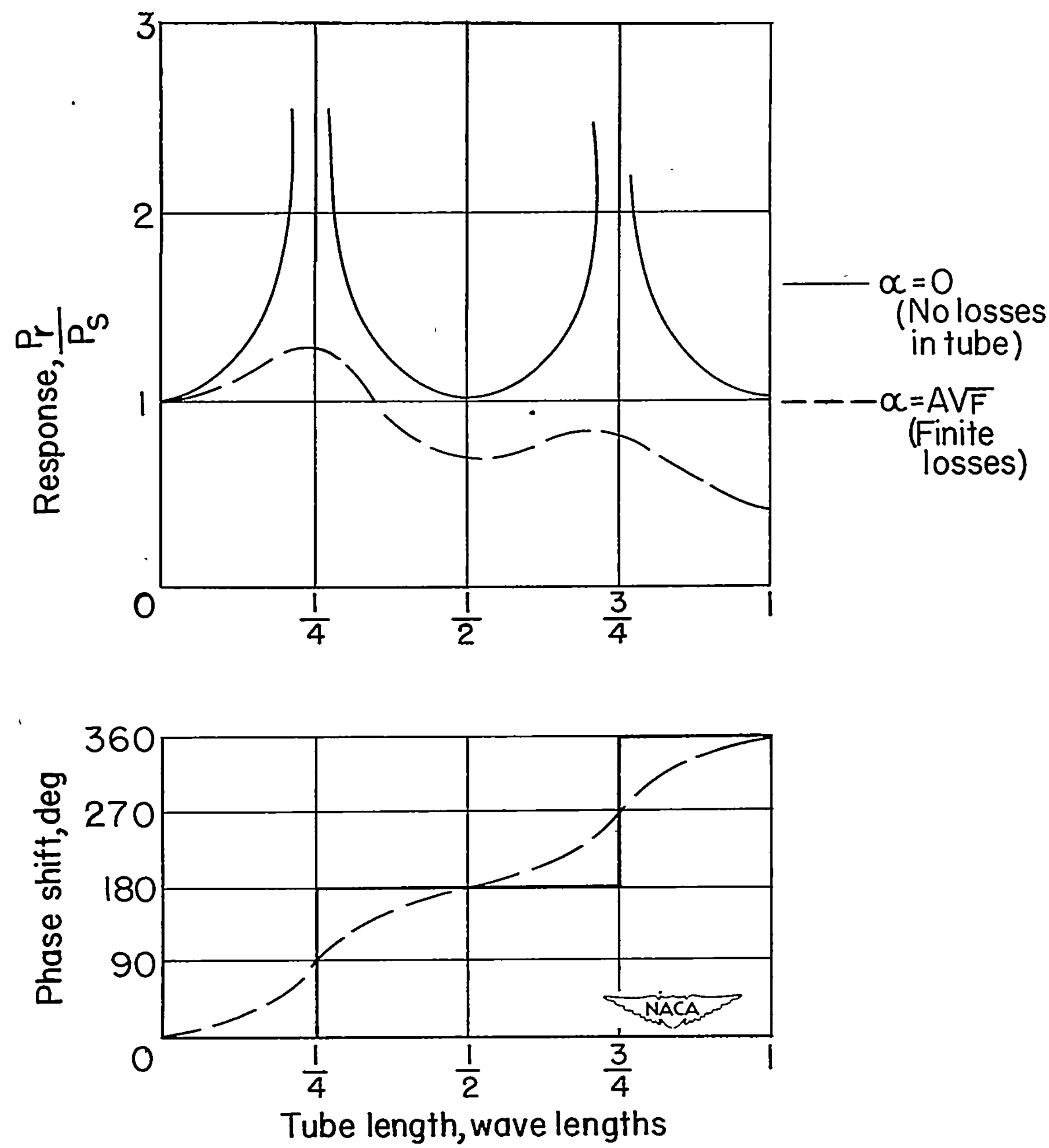


Figure 1.– Response and phase shift in system having negligible instrument volume.

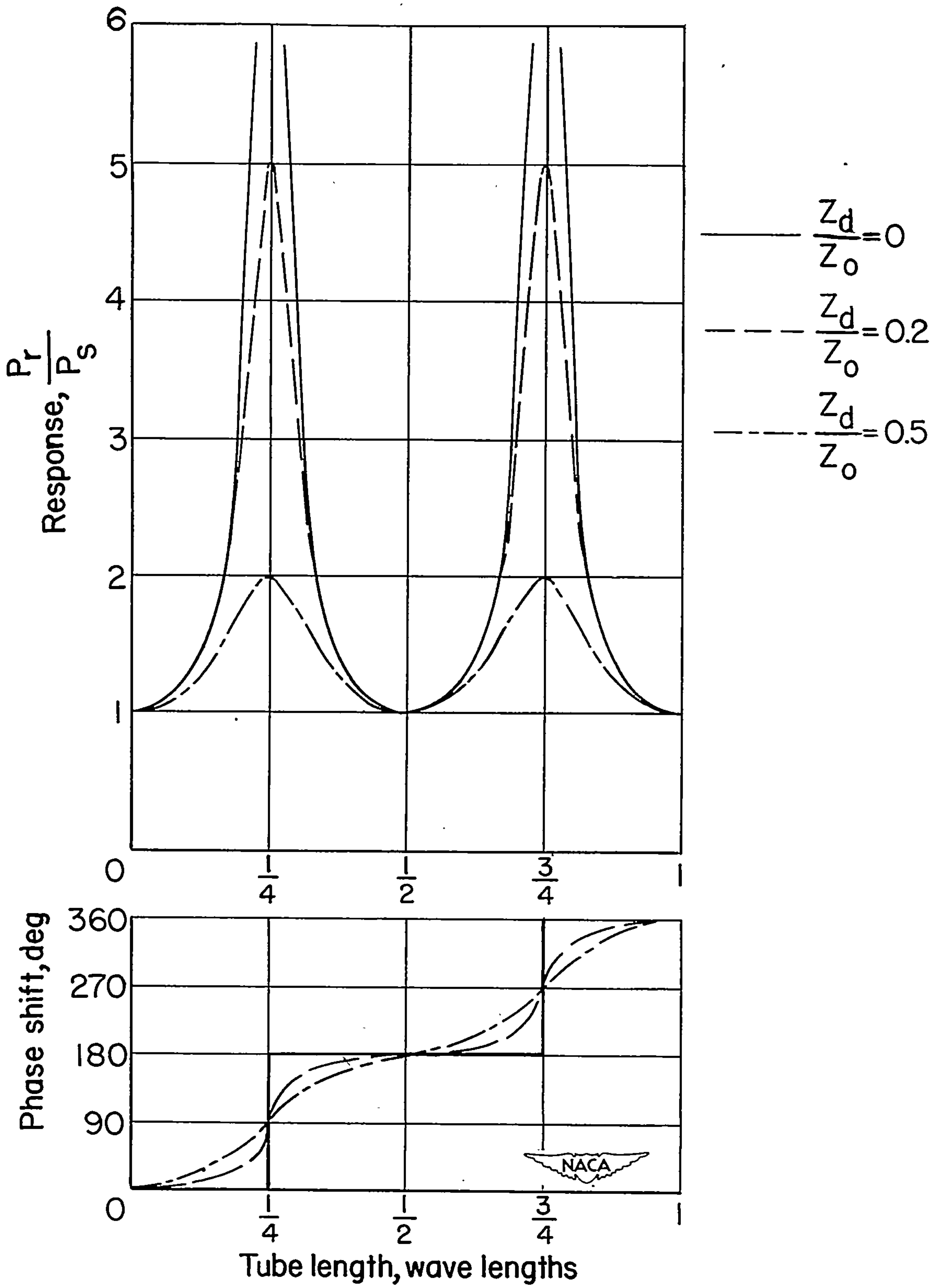


Figure 2.– Response and phase shift in system with various inlet restrictions. $\alpha = 0$.

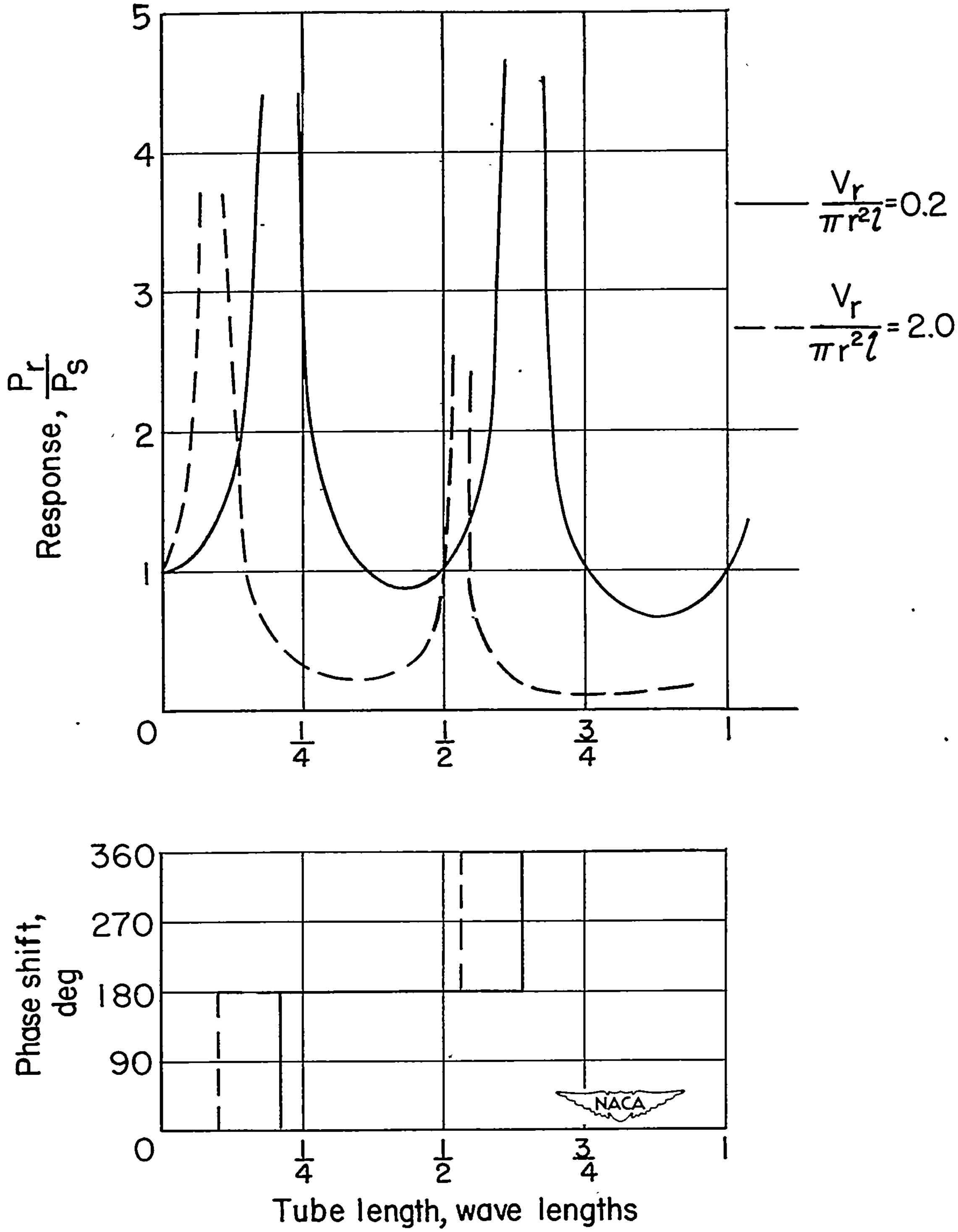


Figure 3.– Response and phase shift in systems with various instrument volumes. $\alpha = 0$.

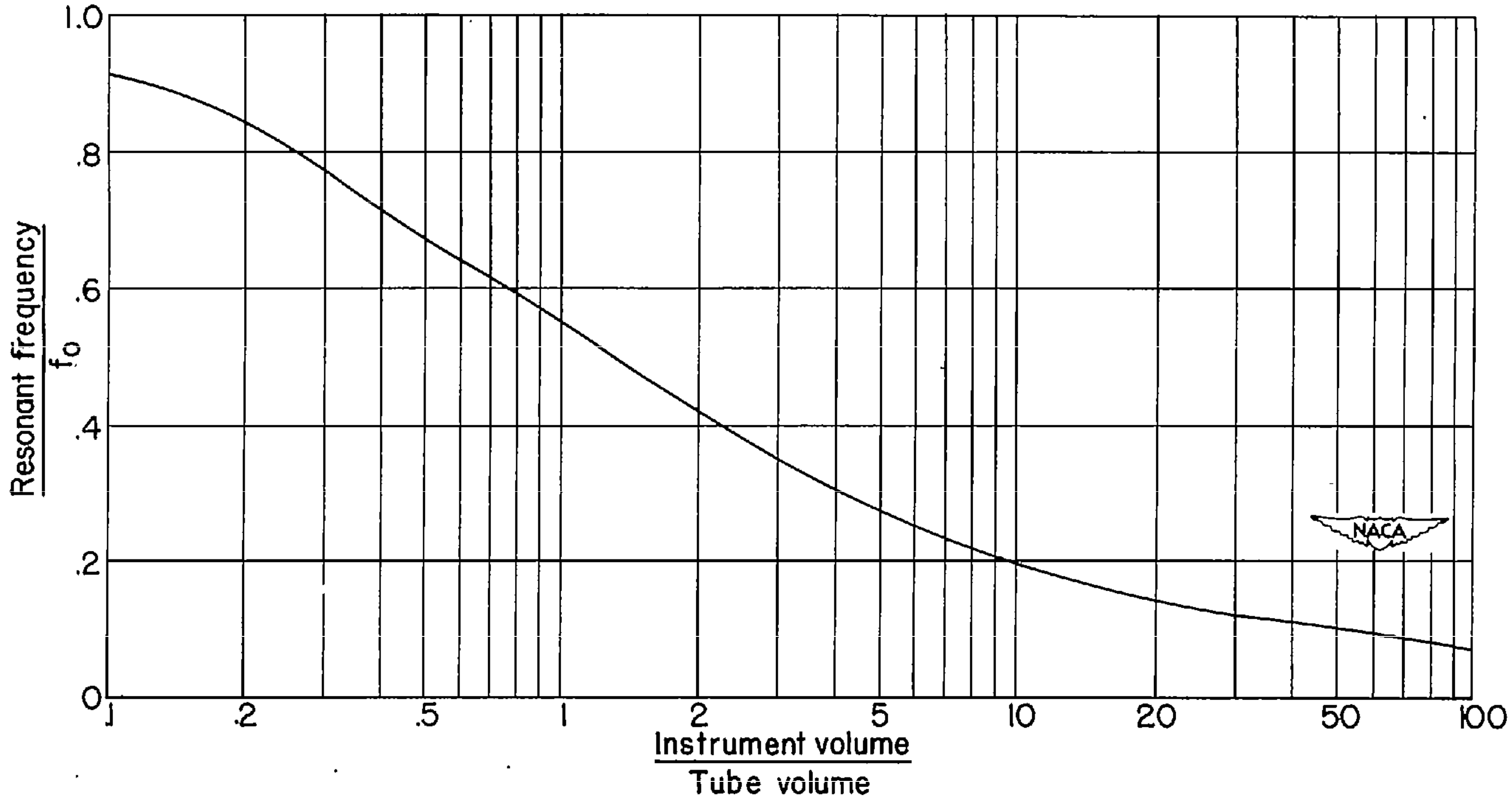


Figure 4.— Resonant frequencies of tubing instrument volume systems. $\alpha = 0$.

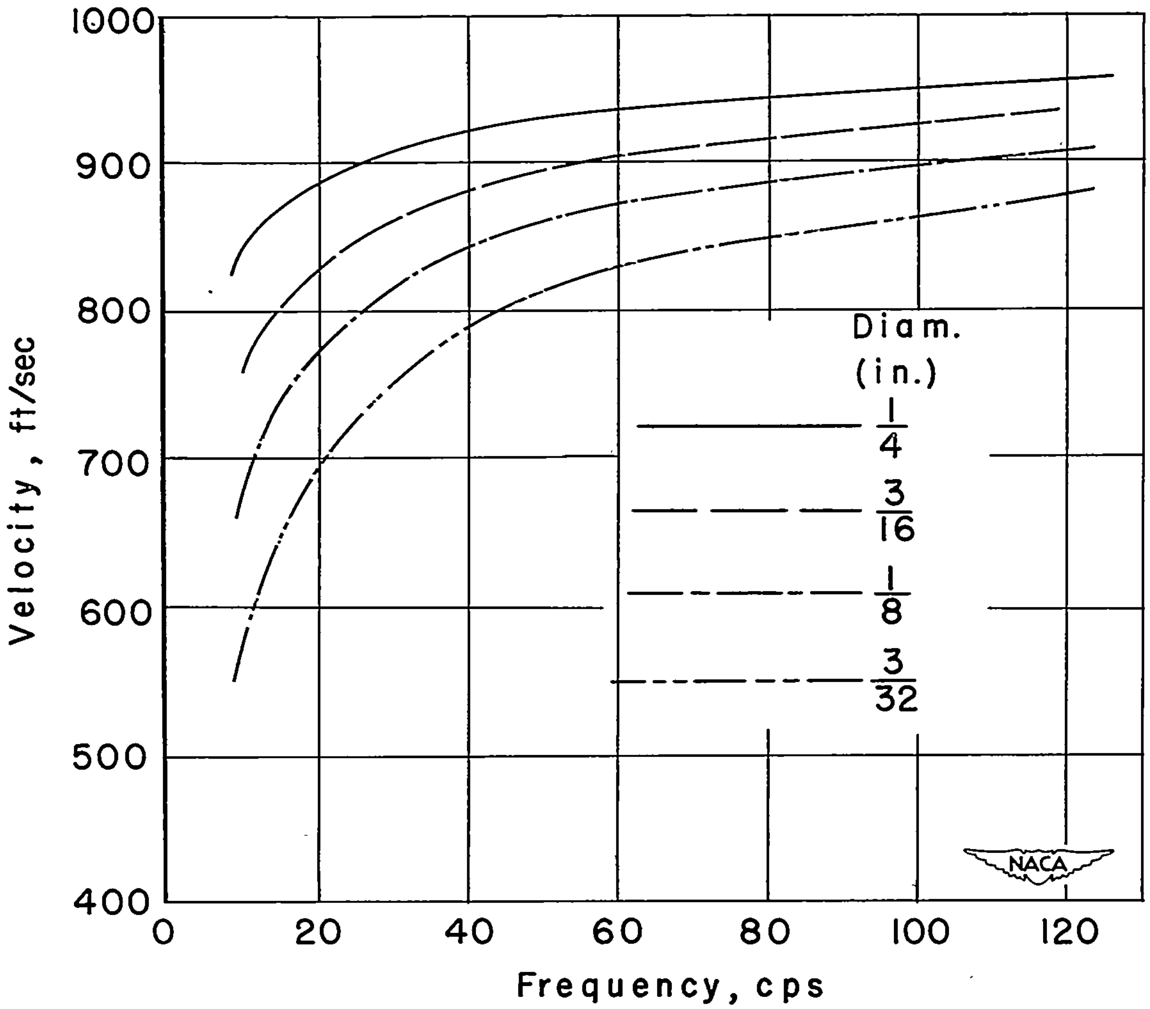


Figure 5.— Velocity of propagation in tube based on Rayleigh formula and free-air velocity of 1000 feet per second.

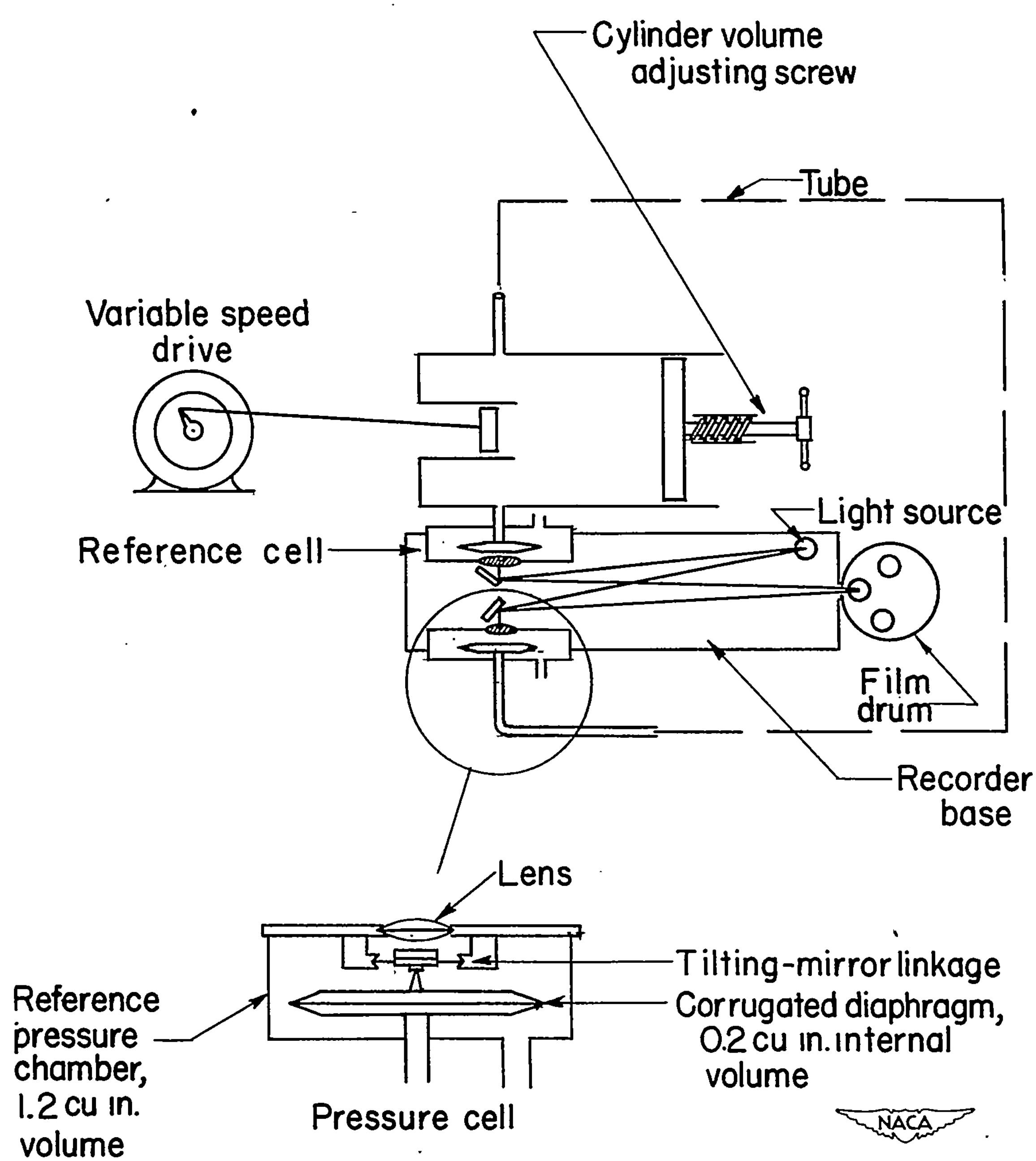


Figure 6.– Schema of test setup and NACA pressure recorders.

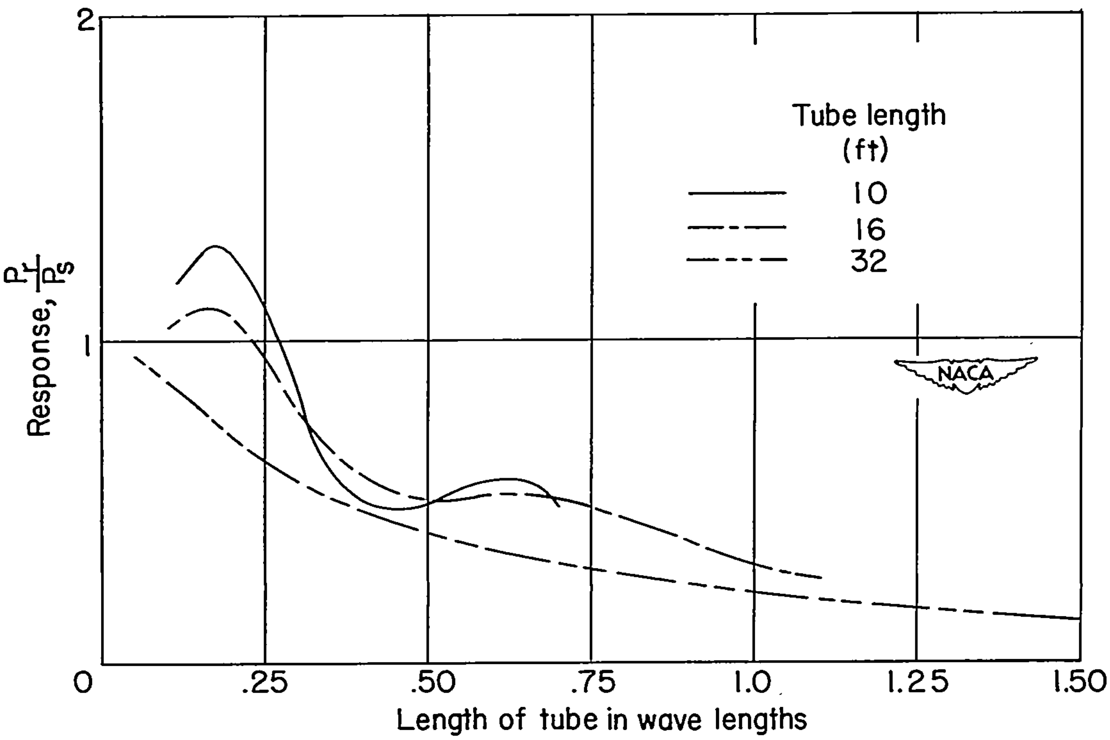


Figure 7.– Response of $\frac{1}{8}$-inch-inside-diameter tubing to sinusoidal pressure variations. Impressed frequency, 0 to 70 cps; pressure amplitudes of ±10 inches of water.

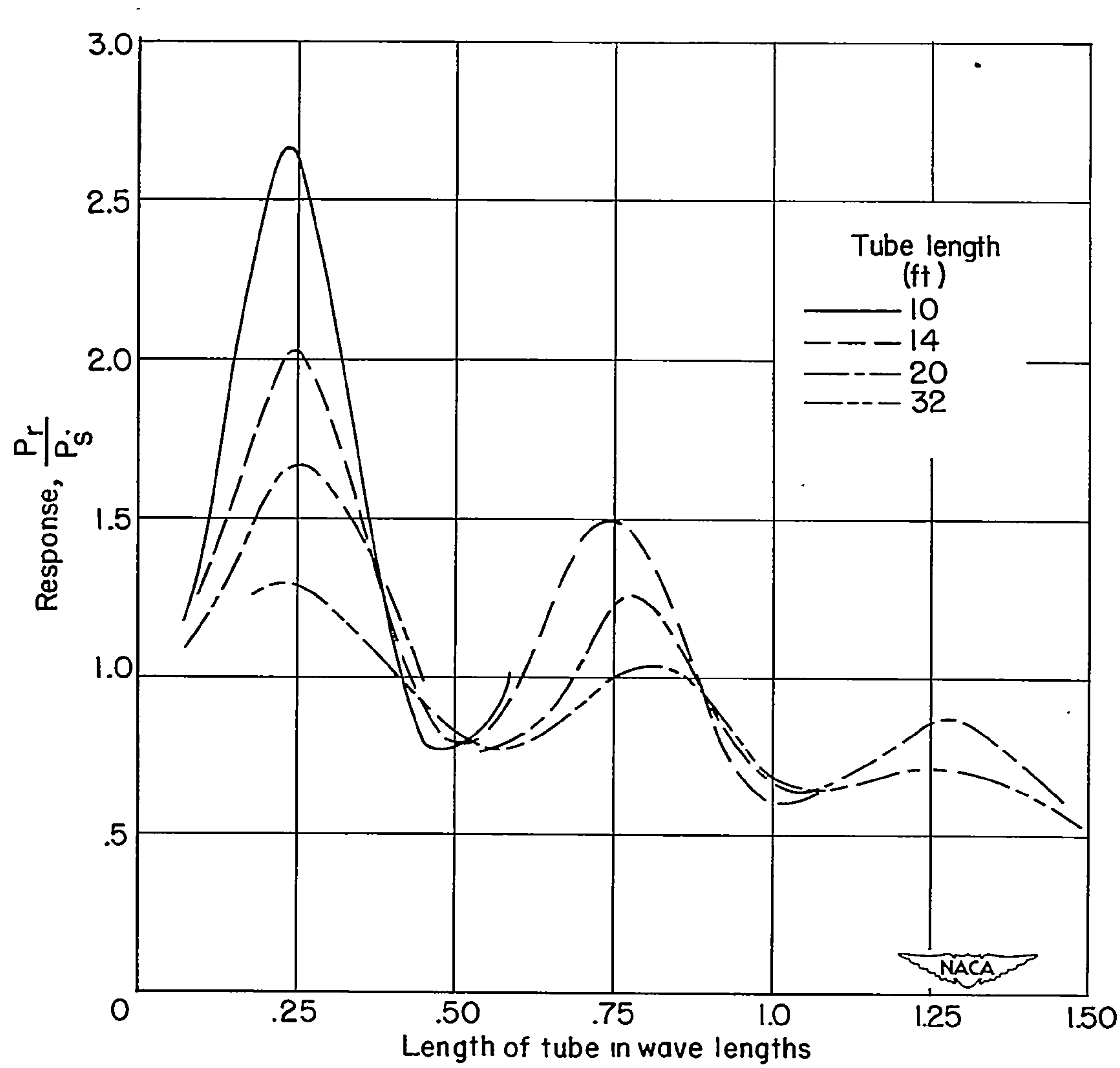


Figure 8.– Response of $\frac{3}{16}$-inch-inside-diameter tubing to sinusoidal pressure variations. Impressed frequency, 0 to 70 cps; pressure amplitudes of ±10 inches of water.

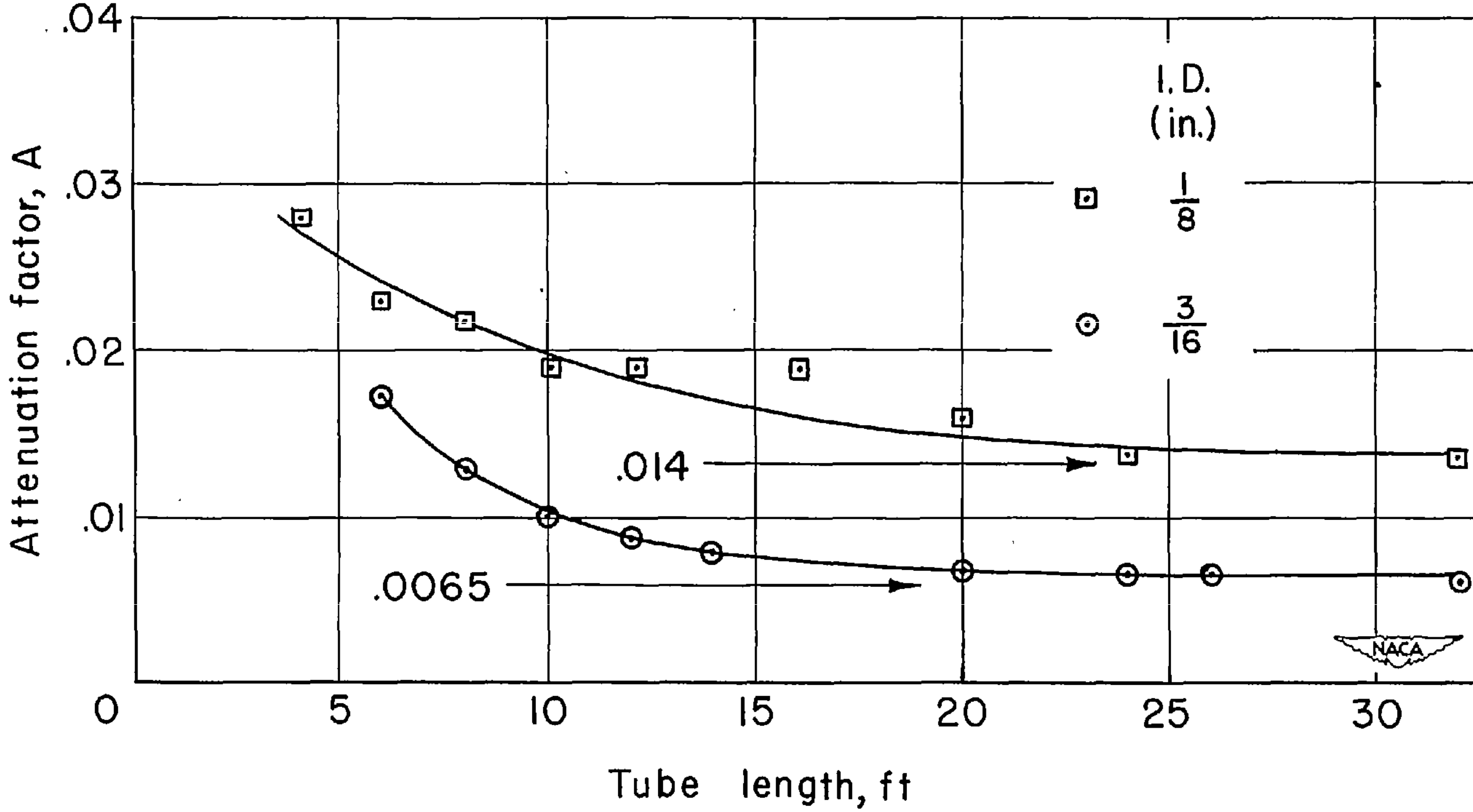


Figure 9.– Attenuation factor for $\frac{3}{16}$-inch-and $\frac{1}{8}$-inch-inside-diameter tubes subjected to sinusoidal pressure amplitudes of ±10 inches of water.

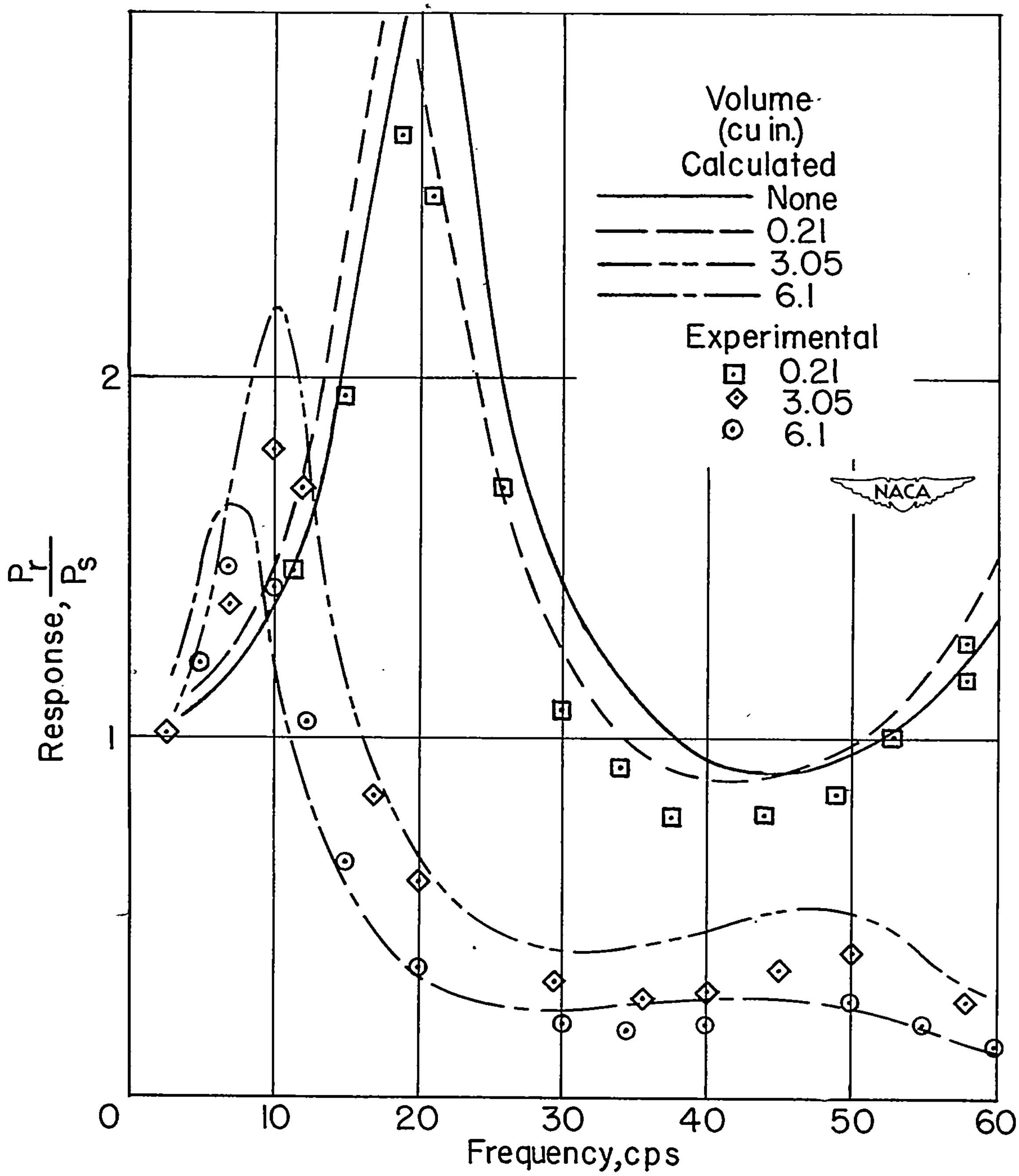


Figure 10.– Response of 10 feet of $\frac{3}{16}$-inch-inside-diameter tubing with various instrument volumes to sinusoidal pressure amplitudes of ±10 inches of water.

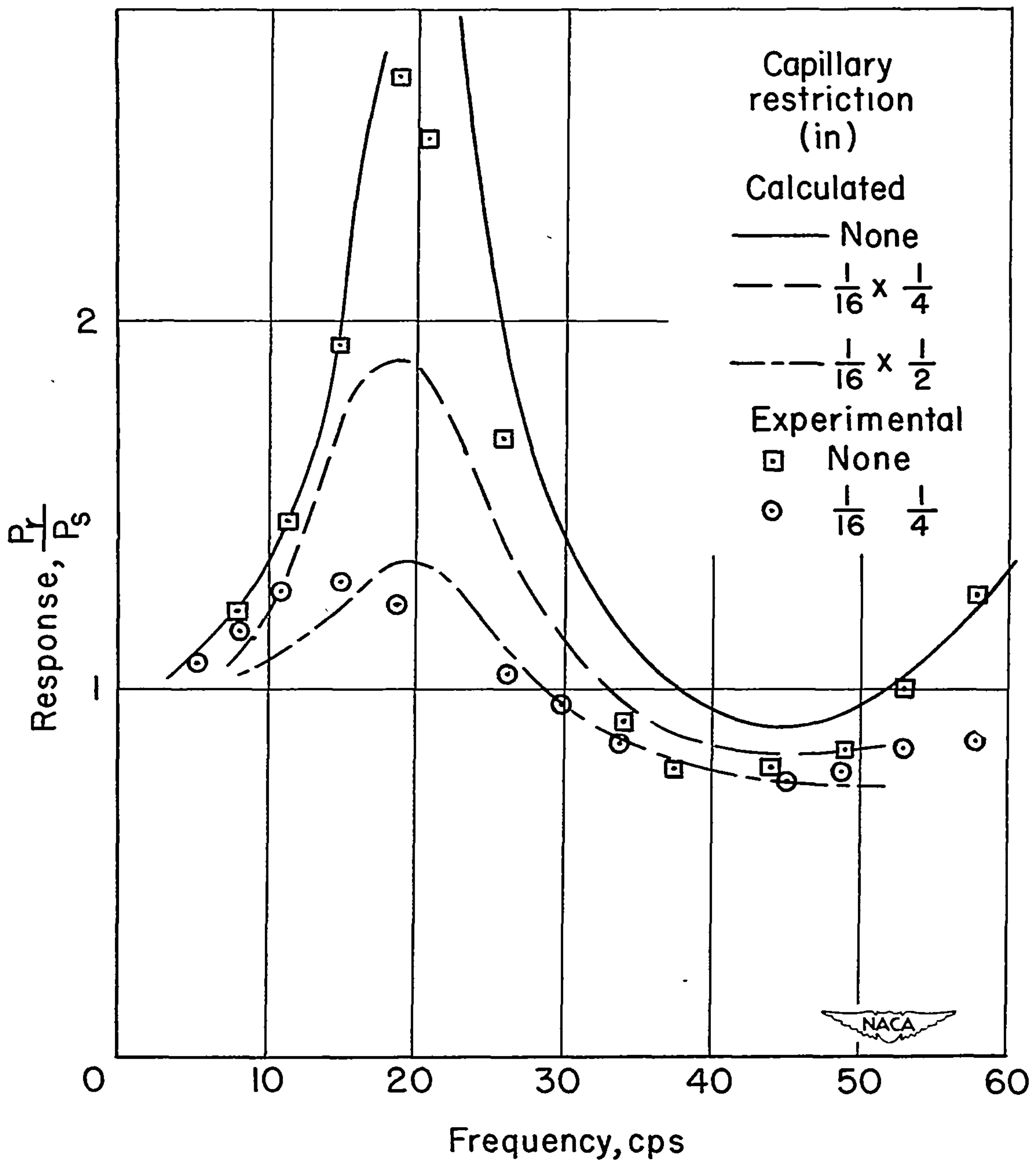
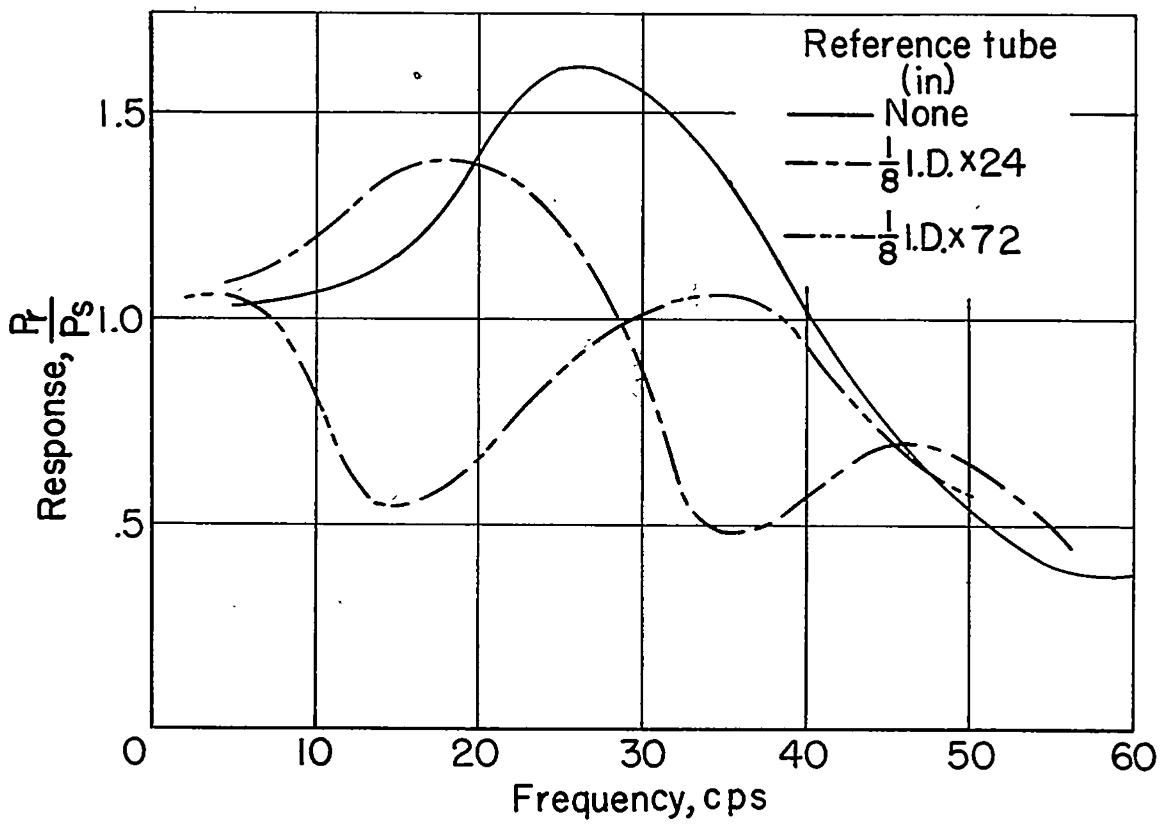


Figure 11.– Response of 10 feet of $\frac{3}{16}$-inch-inside-diameter tubing with and without restrictions to sinusoidal pressure amplitudes of ±10 inches of water.



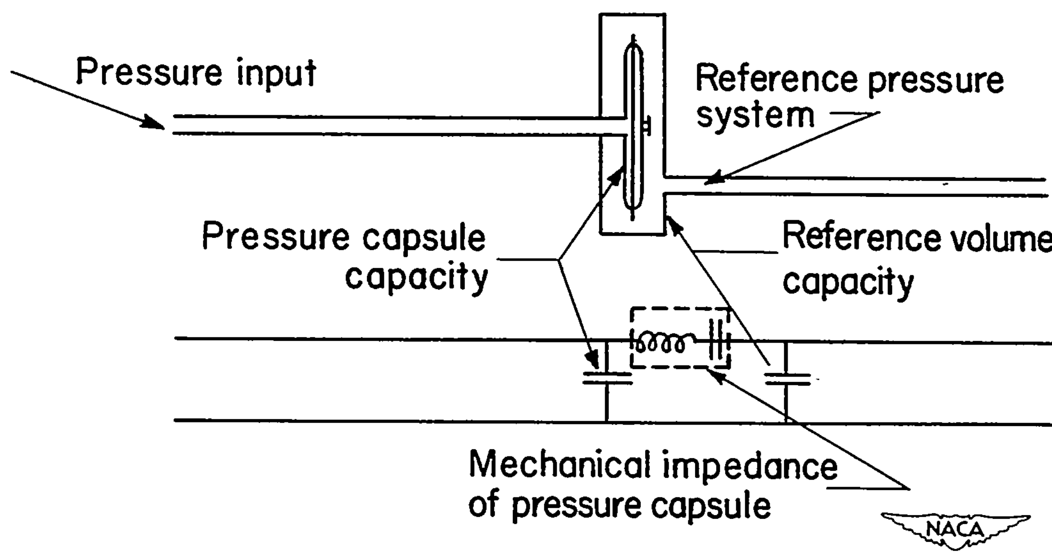


Figure 12.– Response of 2 feet of $\frac{1}{8}$-inch-inside-diameter tubing with various reference pressure tubes to sinusoidal pressure amplitudes of ±10 inches of water.

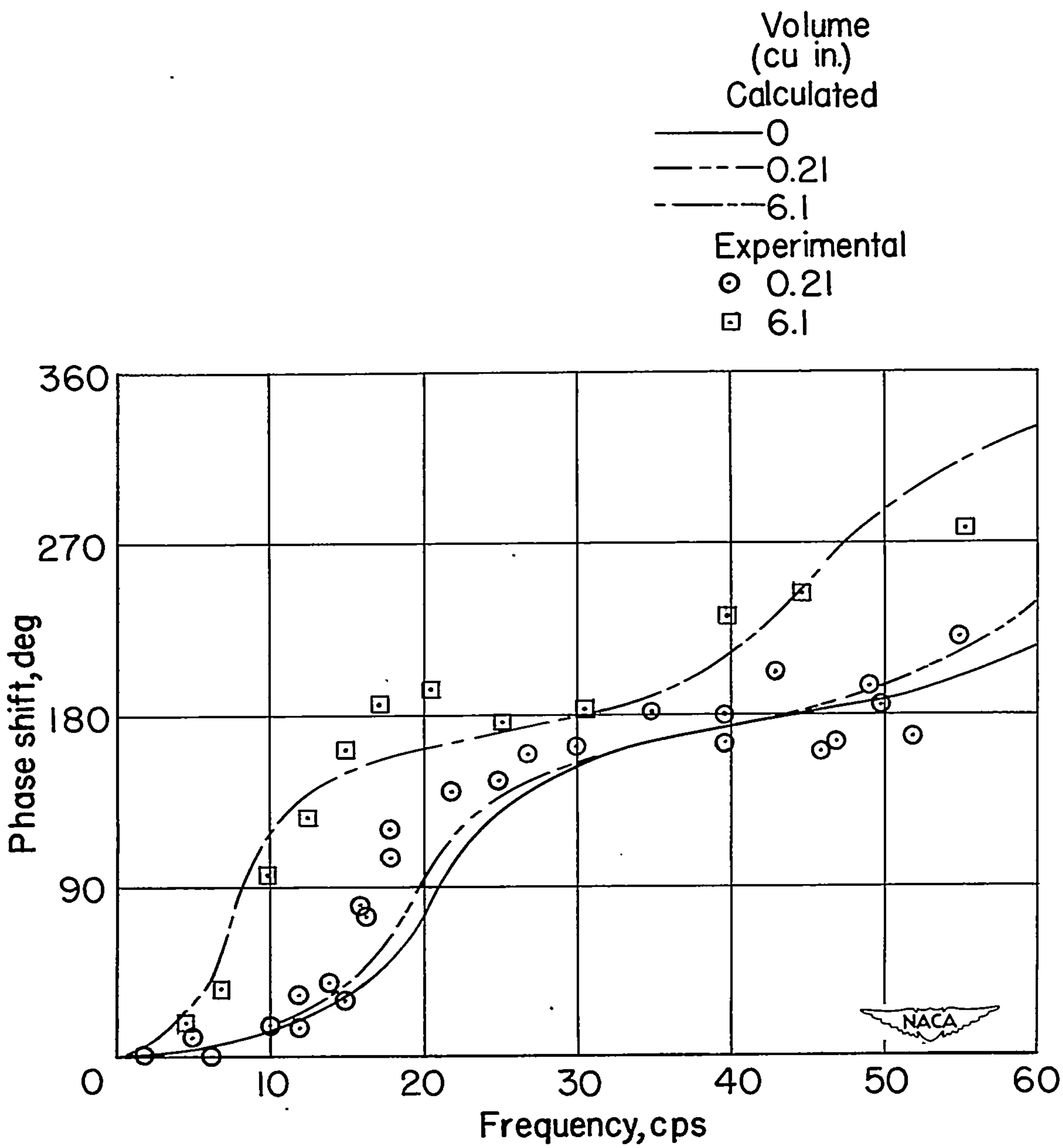


Figure 13.– Lag curves for 10 feet of $\frac{3}{16}$-inch-inside-diameter tubing with various instrument volumes subjected to sinusoidal pressure amplitudes of ±10 inches of water.